MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures ⬚⬚ ⬚⬚⬚ ⬚⬚⬚

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ⬚

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	7	
9	7	
10	8	
11	8	
12	7	
13	9	
14	9	
15	12	
16	9	
17	7	
18	7	
19	8	
S2 Total	98	
S2 Wt ($\times 0.6633$)	65%	

Section Two: Calculator-assumed 65% (98 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (7 marks)

A deputy principal collected data over a 24-hour period on hours of sleep s and hours of mobile phone use h from a random sample of students in their school. The coefficient of determination between the variables was calculated to be 0.85 and the equation of the least-squares line that fitted the data was $\hat{s} = -0.22h + 9.25$.

(a) State what percentage of the variation in the hours of sleep of these students can be explained by the variation in their hours of mobile phone use, and hence assess the strength of the association between these variables. (2 marks)

(b) Determine the correlation coefficient between s and h for this data. (2 marks)

(c) Use the least-squares line to predict the hours of sleep for a student who used their mobile phone for 7.5 hours over a 24-hour period. (1 mark)

(d) Discuss the validity of the prediction in part (b). (2 marks)

Question 9   **(7 marks)**

A harbour has four ferry stations, numbered $1, 2, 3$ and 4. A ferry can only be caught from 1 to 3, from 2 to 1, from 3 to 2, from 3 to 4 and from 4 to 3. There is also a harbour sightseeing ferry that starts and finishes at station 4.

(a)   Draw directed graph G to represent the above information.   (2 marks)

(b)   Construct M, the adjacency matrix for G.   (2 marks)

(c)   Lolo bought an all-day ferry ticket, caught some ferries and ended up at station 1. Her journey formed a walk of length 5 in graph G. Form an appropriate multistage matrix and use it to explain which station(s) she could **not** have started from.   (3 marks)

Question 10 **(8 marks)**

The weight of fish caught by anglers on the first and second day of a 7-day angling competition was 17.5 and 19.6 kg respectively.

Let the weight of fish caught on day n of the competition be T_n kg.

(a) Assume that the daily weights form an arithmetic sequence.

    (i) What weight of fish was caught on the third day of the competition? (2 marks)

    (ii) Deduce a rule that models the weight of fish caught on the nth day of the competition. (1 mark)

    (iii) What weight of fish was caught on the last day of the competition? (1 mark)

(b) Assume that the daily weights form a geometric sequence.

    (i) Determine a recursive rule to model the daily weight of fish caught. (2 marks)

    (ii) To the nearest 0.1 kilogram, what weight of fish was caught on the last day of the competition? (2 marks)

Question 11 **(8 marks)**

The balance a_n of savings account A after n monthly payments of $50 have been made into it can be modelled using the following recurrence relation:

$$a_{n+1} = 1.0084a_n + 50, \qquad a_0 = 300.$$

Savings account A has an interest rate of 0.84% per month.

(a)   State the balance of the savings account A before any monthly payments were made.

(1 mark)

(b)   Determine the balance of the savings account A after 15 monthly payments have been made.   (2 marks)

(c)   Determine k, the number of monthly payments that are required for the balance of savings account A to first exceed $4000, and state the value of a_k.   (2 marks)

The recurrence relation $b_{n+1} = 1.0052b_n + 44$, $b_0 = 500$, models the balance b_n of a similar savings account B after n monthly payments of x have been made into it.

(d)   State the value of x, the monthly payment made into savings account B.   (1 mark)

The difference in the balances of savings accounts A and B is least after y monthly payments have been made into each account.

(e)   Determine the value of y and the difference between the balances of the accounts at this time.   (2 marks)

Question 12 **(7 marks)**

A software company is developing two smartphone apps called Sweety and Pics to estimate the percentage sugar content of common foodstuffs from photos taken by the phone's camera.

The following table shows estimates made using the Sweety app and the actual percentage sugar content for eight foodstuffs, where $r_{xy} = 0.799$.

Foodstuff	A	B	C	D	E	F	G	H
Estimate x	29	7	44	15	22	31	11	35
Actual y	57	4	58	29	43	31	32	45

(a) Using the estimate x as the explanatory variable, determine the equation of the least-squares line to model the linear relationship between x and y. (1 mark)

When the same eight foodstuffs were used to trial the Pics app, the equation of the least-squares line to predict the actual sugar content was $\hat{y} = 0.88x - 4.08$ and $r_{xy} = 0.91$.

(b) Does the Pics app tend to over or underestimate the actual sugar content? Justify your answer. (2 marks)

Sweety and Pics estimated the sugar content of a ninth foodstuff as 33% and 41% respectively.

(c) Use the estimate from each app to predict the actual sugar content of the ninth foodstuff and explain which prediction you have more confidence in. (4 marks)

Question 13   **(9 marks)**

A river runs through a town, separating the northern region N from the southern region S. The river also splits, forming islands W and E, and six bridges cross the river at locations shown on the sketch map below.
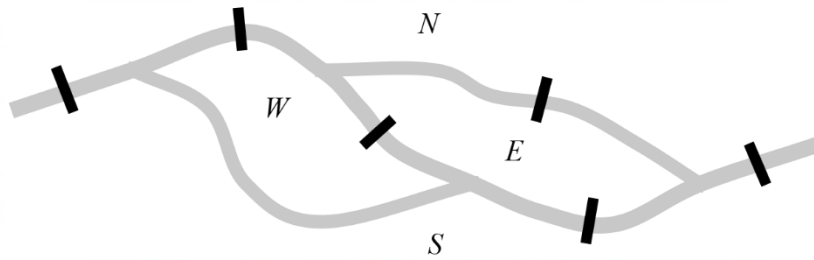
(a) Use the information shown in the sketch map to draw a graph in the plane, in which vertices and edges represent the regions and the bridges respectively.   (3 marks)

(b) A tourist, staying in a hotel on island E, wants to take a morning run that starts from their hotel and crosses every bridge in the town once. They don't mind where their run ends. If possible, describe how they can do this. If not possible, explain why not.   (2 marks)

(c) Planning is underway for a seventh bridge in the town. Explain how your answer to part (b) would change if the new bridge was a second river crossing between

     (i)   regions N and W.   (2 marks)
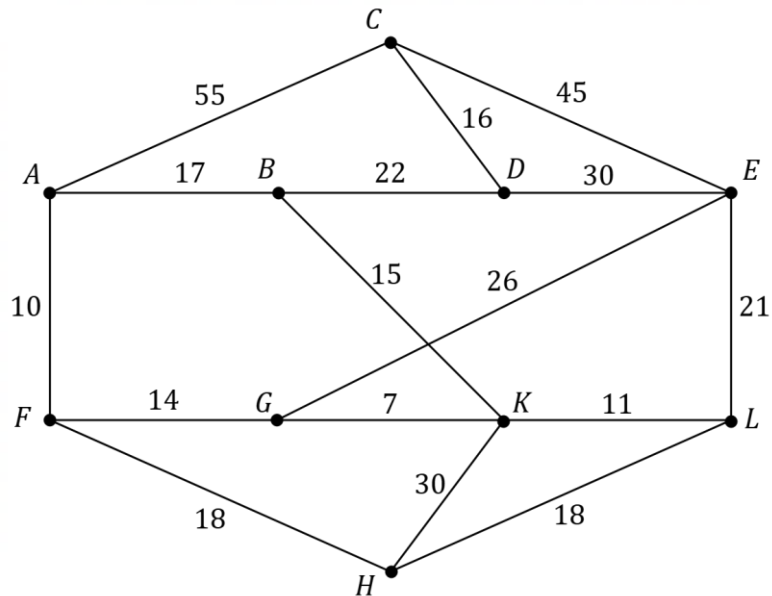
     (ii)   regions S and E.   (2 marks)

Question 14   **(9 marks)**

A company specialises in transporting sculptures, charging an amount equal to the cheapest path that exists between any pair of cities in the network shown below. Each edge weight represents the cost, in hundreds of dollars, to transport a sculpture along that edge.
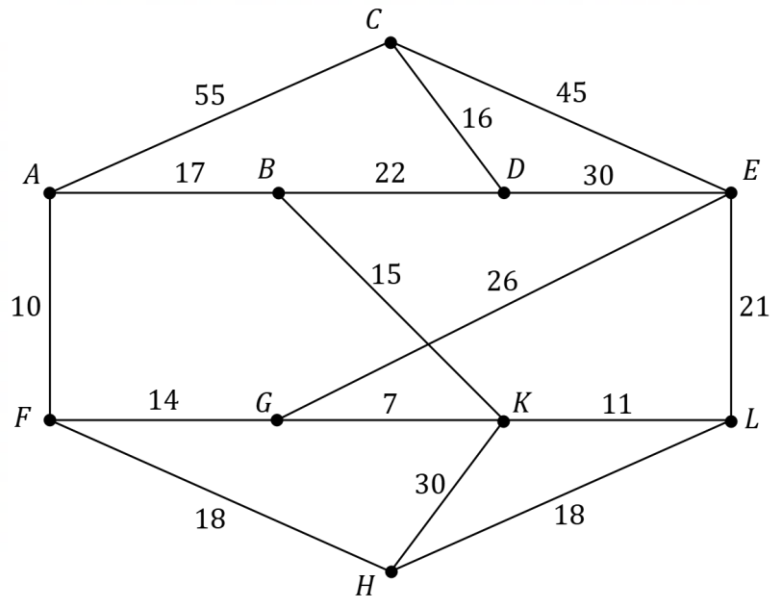

(a)   Determine how much the company will charge, and the associated path, to transport a sculpture between cities

     (i) A and E.   (3 marks)

     (ii) C and H.   (3 marks)

(b)   Determine the increase in the amount charged to transport a sculpture between cities C
       and H if the company raised every cost shown on the network by $300. A copy of the
       original graph has been provided below.   (3 marks)

Question 15 **(12 marks)**

The number of years of experience t and the current salary s, in thousands of dollars, is shown in the table below for a sample of librarians who belong to a professional association.

t (years)	4	6	8	3	7	10	2	11	5
s ($000)	83	82	87	72	89	97	67	105	82

(a) Use your calculator to construct a scatterplot of the data with t as the explanatory variable and hence describe the nature of the relationship between the variables. (2 marks)

(b) Determine the equation of the least-squares line that can be used to predict s from t and state the correlation coefficient. (3 marks)

(c) Interpret, in the context of this question,

(i) the slope of the least-squares line. (2 marks)

(ii) the intercept of the least-squares line. (1 mark)

(d)   What percentage of the variation in current salary of these librarians can be explained by the variation in number of years of experience?   (1 mark)

(e)   Another librarian who belonged to the same professional association has 15 years of experience.

   (i)   Use the least-squares line to predict the salary of this librarian.   (1 mark)

   (ii)   Is the prediction in part (e)(i) reliable? Justify your response.   (2 marks)

See next page
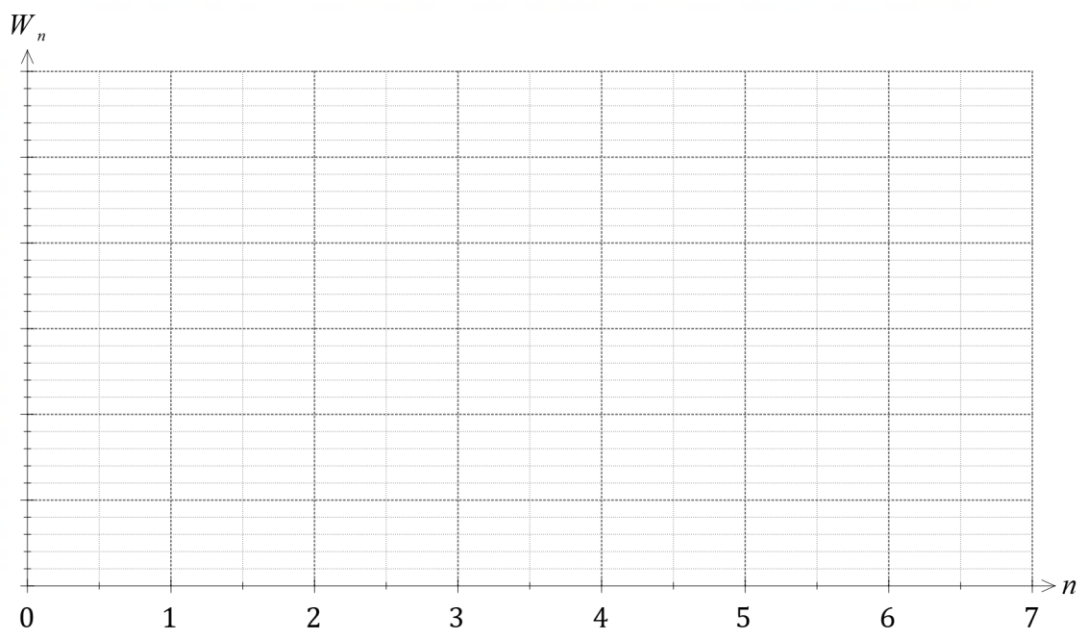
Question 16 **(9 marks)**

The volume of oil wasted, V_n litres, during the nth trial of a new refining process is given by

$$V_{n+1} = -0.7V_n + 34, \qquad V_1 = 30.$$

(a) Use the recurrence relation to complete the following table. (2 marks)

n	1	2	3	4	5	6	7
V_n (L)	30						21.2

(b) Add a suitable scale to the vertical axis and then display the volume wasted in each trial on the axes below. (3 marks)

(c) Will the volume wasted in the twelfth trial be more or less than in the tenth trial? Explain how you can answer this question without calculating W_{10} or W_{12}. (2 marks)

(d) Describe how the volume wasted changes in the long term. (2 marks)
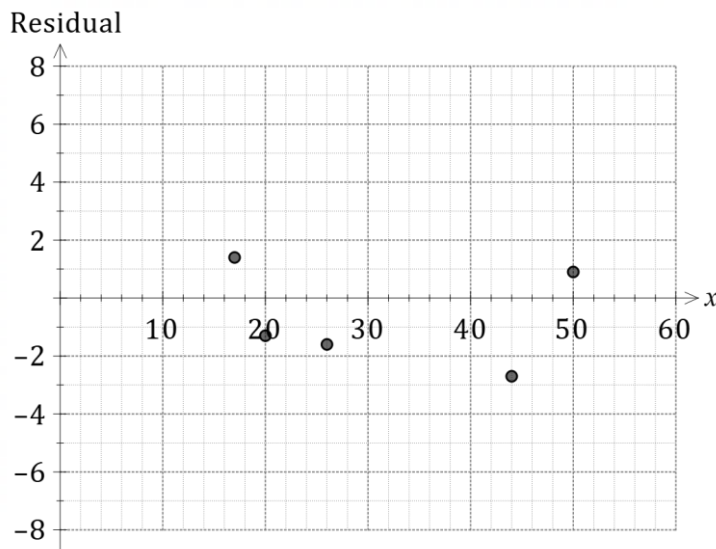
Question 17 **(7 marks)**

The data in the following table shows x, the size of a country's iron ore stockpile in millions of tonnes and y, the price paid by that country to import iron ore in dollars per tonne.

x (Mt)	11	17	20	26	32	38	44	50	59
y ($)	126	112	105	96	84	75	69	64	57

The data has a correlation coefficient of -0.982 and the equation of the least-squares line is $\hat{y} = 135.1 - 1.44x$.

(a) Show how to use the point $(20, 105)$ to calculate its residual of -1.3. (2 marks)

(b) Complete the residual plot below. (3 marks)

Residual

(c) Based on the residual plot, is it appropriate to fit a linear model to the data? Justify your answer. (2 marks)

Question 18 **(7 marks)**

Galatea has created graph G to represent a communications network within the company that she works for. The graph has $2x - 5$ faces and at least one vertex of degree x.

(a) Give two reasons why the value of x cannot be 2.5. (2 marks)

Three more properties of G are that it is a connected planar graph, it has $2x + 3$ edges and it has $2x - 8$ vertices.

(b) Determine the value of x. (3 marks)

(c) State, with justification, whether G is Eulerian. (2 marks)

See next page

Question 19 **(8 marks)**

Farah suspected a small dam on her farm had a leak and began to monitor the depth of water every hour. The depth of water was initially 15.7 m and subsequent readings indicated it was decreasing by 0.6% every hour.

Let d be the depth of water in the dam t hours after monitoring began.

(a)   Write an exponential equation in the form $d = ar^t$ to model the depth of water in the dam.

(2 marks)

(b)   Determine the depth of water in the dam 3 days after monitoring began.   (2 marks)

The leak was caused by a damaged valve. At the instant Farah fixed the valve, exactly 4 days after monitoring began, heavy rainfall in the region caused the depth of water in the dam to abruptly change from decreasing by 0.6% to increasing by 1.2% every hour.

(c)   Determine the depth of water in the dam 6 days after monitoring began.   (4 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____